EXPERIMENTAL STUDY OF CIRCULAR COUETTE FLOW IN A COPLANAR MAGNETIC FIELD

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We present and discuss results from an experimental investigation of the hydrodynamic parameters of circular Couette flow in a coplanar magnetic field: the velocity profiles, the oscillograms showing the pulsations in total and static mercury heads, and the relationship between the coefficient of friction at the wall and the dimensionless Reynolds and Hartmann numbers.

Particular attention should be devoted in magnetohydrodynamics to the effect of magnetic fields on the turbulent flows of electrically conductive fluids, to reduce the intensity of turbulence.

Magnetohydrodynamics suggests a possibility of reducing turbulence that is qualitatively different from that of conventional hydrodynamics, i.e., by suppressing the turbulent pulsations in velocity.

Theoretical studies along these lines are exceedingly complex and at the contemporary level the role of experimental investigations is decisive.

Both here and abroad, a number of experimental installations have been designed during the past decade [1], and these have been used to perform various investigations of fluid flow in transverse and longitudinal magnetic fields. To suppress velocity and pressure pulsations, the "purer" approach is a study of the effect of the longitudinal magnetic field, since in the case of a transverse magnetic field the familiar Hartmann effect increases the frictional resistance of the fluid against the solid wall and complicates analysis of the integral flow characteristics.

However, in all of the installations (with a longitudinal magnetic field) with which we are familiar, it was exclusively flow in tubes that was studied, and this is apparently explained by the great difficulty in the design of installations with another type of flow (for example, the streamlining of a plate) in a longitudinal field.

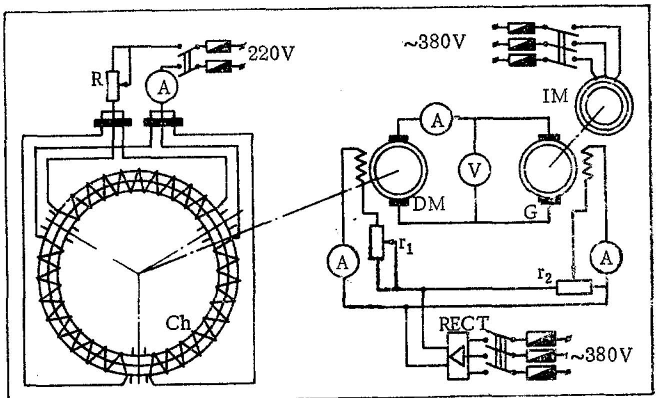

Fig. 1. Diagram of the experimental installation: Ch) annular channel with rotating cylinder and toroidal electromagnet winding; R) RM-1671 rheostat to regulate current in electromagnet winding; IM) induction motor; G) dc generator; DM) drive motor; r_1 and r_2) rheostats to regulate revolutions of the drive motor; Rect) rectifier.

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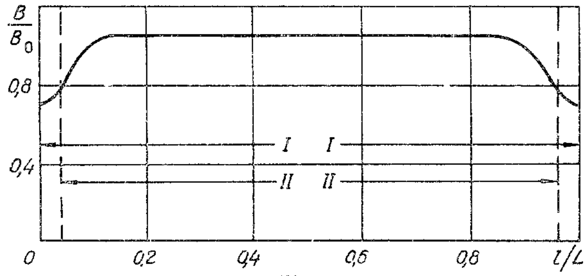

Fig. 2

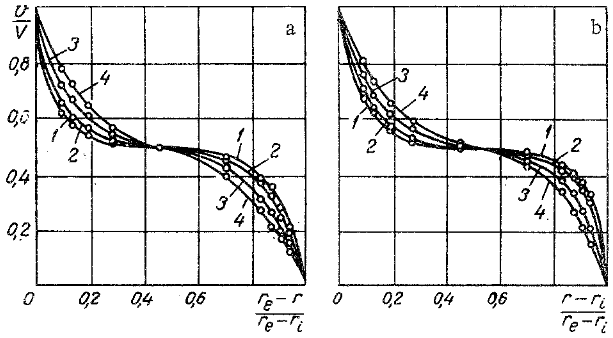

Fig. 3

Fig. 2. Distribution of magnetic induction in the channel (within the confines of one of the 3 sections of the electromagnet): I–I) flow sampling planes; II–II) boundaries of the electromagnet winding segments; B/B_0) magnetic induction with respect to its mean integral value; l/L) relative coordinate (with respect to the length of the electromagnet segment).

Fig. 3. Velocity profiles in the clearances [a) inside clearance; b) outside clearance] between the channel walls and the rotating cylinder for $Re = 5.6 \cdot 10^3$, $Ta = 1.0 \cdot 10^3$; v/V is the relative velocity in the clearances; r_e is the radius of the outside (external) cylinder; r_i is the radius of the inside cylinder; $r_e - r_i = \delta$ is the clearance between the rotating and fixed cylinders [1) M = 0, 2) 39; 3) 64; 4) 84].

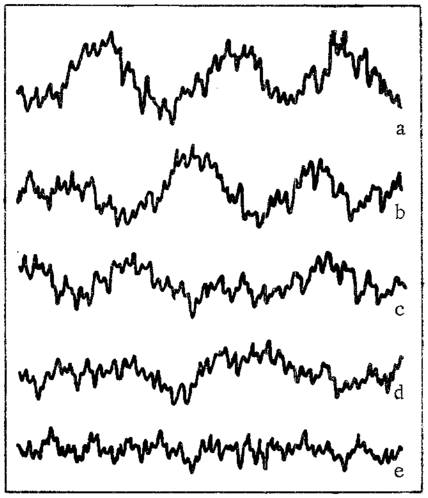

Fig. 4. Fluctuation in total head for $Re = 5.5 \cdot 10^3$, time 0.3 sec: a) M = 0; b) 17; c) 39; d) 64; e) 84.

Certain theoretical investigations show that in various types of flow the effect of the magnetic field may be extremely diverse. Thus, for example, the investigations of Edmonds [2] into the stability of circular Couette flow in a coplanar circular (i.e., essentially longitudinal) magnetic field led him to the conclusion that the field exerts considerably less influence on the stability of this kind of flow than on the stability of the flow of a fluid in a flat channel (following the Stewart study). On the other hand, it is rather difficult to provide a physical basis for this difference. In this connection, there is an urgent need for experimental investigations into the effect of a magnetic field on this kind of flow.

An experimental installation was developed and constructed to operate with liquid metal for Couette flow in a longitudinal magnetic field. The installation is an annular channel with an electrically conductive liquid (mercury), and on the inside of this channel there is a cylinder which is rotated by means of a mechanical drive actuated by an electric motor powered from a generator–motor system (Fig. 1). As the cylinder rotates, a circular Couette flow is set up between the cylinder and the inside surfaces of the channel walls; this provided for the simultaneous simulation of two cases of flow: a) with a rotating inside cylinder and a fixed outside cylinder (in the clearance that is on the outside, relative to the cylinder) and b) with a fixed inside cylinder and a rotating outside cylinder (in the inside clearance). The clearances exhibited a width that was 1.6% of the mean cylinder diameter. The circular magnetic field was produced by the electric current flowing in the toroidal winding situated at the surface of the annular channel. The installation permitted experiments within the following ranges: $Re = 5 \cdot 10^3$–$3 \cdot 10^5$, M = 0–84, $Ta = 10^3$–$5 \cdot 10^4$. Samplings of the flow were taken at 3 points between the segments of the toroidal winding.

The purpose of the investigation was to determine the quantitative relationships governing the effect of a longitudinal magnetic field on the hydrodynamic flow parameters for a conducting fluid (at various Re numbers) in the space between a moving and a fixed wall: 1) the distribution of the time-averaged velocities; 2) the level of pulsations in velocity and pressure at various points of the lateral cross section of the flow; and 3) the coefficient of liquid friction at the wall.

For the first item, we measured the total and static mercury heads. The probes were fashioned from the injection elements of Pitot and Prandtl tubes, which were moved across the flow by means of

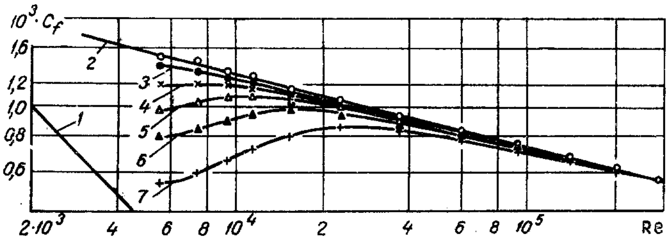

Fig. 5. Experimental determination of the friction factor C_f at the wall as a function of the Reynolds number for various values of the Hartmann number: 1) $C_{f_1} = 2/Re$ is the friction factor in a laminar flow regime; 2) $C_{f_0} = 0.013\ Re^{-0.25}$ is the friction factor in a turbulent flow regime, M = 0; 3) M = 17; 4) 39; 5) 52; 6) 64; 7) 84.

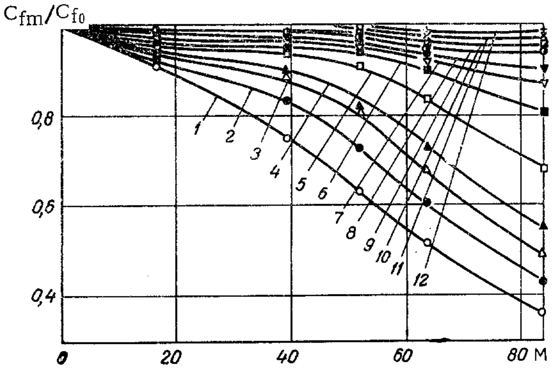

Fig. 6. Reduction in the friction factor at the wall as a function of the Hartmann number for various values of the Reynolds number: 1) Re = $5.5 \cdot 10^3$; 2) $7.3 \cdot 10^3$; 3) $9.3 \cdot 10^3$; 4) $1.12 \cdot 10^4$; 5) $1.53 \cdot 10^4$; 6) $2.3 \cdot 10^4$; 7) $3.7 \cdot 10^4$; 8) $6.0 \cdot 10^4$; 9) $9.3 \cdot 10^4$; 10) $1.40 \cdot 10^5$; 11) $2.1 \cdot 10^5$; 12) $2.8 \cdot 10^5$.

micropositioning devices. The heads were measured with a two-liquid piezometer whose upper portion was slanted with a variable angle of inclination ranging from 10° to 90°, thus producing a gain of 50 at small pressure differences. The mirror scale and the viewers made it possible to determine the pressure differences with an error of ±2.0 mm Hg, which corresponds to the accuracy of velocity determination with an error of ±3-1% for Re = $5 \cdot 10^3$–$8 \cdot 10^3$; when Re > 10^4 the error did not exceed ±0.2%.

To trace the direct effect of the magnetic field on the fluctuations in velocity and pressure, using the same probes and titanate–zirconium piezoelectric gauges, we recorded the fluctuations in the total and static heads (at various points on the lateral cross section of the flow). The signals were recorded with an S1-19 oscillograph with two preamplifiers at the input.

For the third of the above purposes of the investigation, we measured the frictional force of the mercury against the channel walls (the fixed cylinders) by means of strain gauges attached to the beams from which the annular channel was suspended.

The temperature of the mercury throughout the tests varied from 20° to 25°; the Chromel–Copel thermocouples with a PP-1 potentiometer regulated the temperature with an error of ±0.5°.

The magnetic field within the channel was calibrated with a specially prepared coil and an M19 type milliweber-meter, with control measurements performed at accessible points of the magnetic circuit with an IMI-3 instrument. The distribution of the magnetic induction in the channel is shown in Fig. 2. In calculating the Hartmann numbers, we used the mean integral value of the induction (B_0). From the measured total and static heads of the liquid we calculated the velocity values in the clearances between the moving and fixed cylinder walls. Figure 3 shows the velocity profiles within the inside and outside clearances for Re = $5.6 \cdot 10^3$ and for various Hartmann numbers.

Analysis of these velocity profiles permits us to draw the following conclusion: the effect of the longitudinal magnetic field on the turbulent motion of the mercury for Re = $5 \cdot 10^{-3}$–10^4 leads to a noticeable reduction in the velocity gradient at the wall; however, as the Reynolds number increases, this effect becomes less pronounced and when Re = $3 \cdot 10^5$ we find no effect exerted by the magnetic field on the distribution of the velocities, within the limits of experimental error. This is a natural result, because if the Stewart number were to reach 1.4 at low velocities, it would amount only to 0.02 at the maximum velocities.

Similar results were found in oscillographing the fluctuations in total and static heads. Figure 4 shows typical oscillograms of the fluctuations in the total mercury head, and from the these we can see that when Re = $5.5 \cdot 10^3$ the amplitudes of the fluctuations diminish at M = 84 by a factor of approximately three.

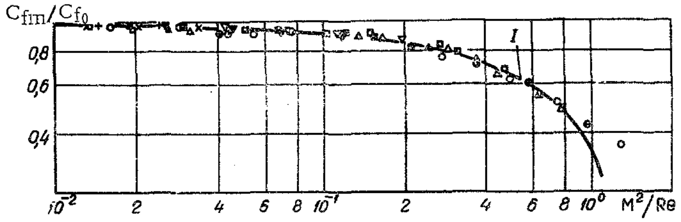

Fig. 7. Relative friction factor as a function of the Stewart number: I) approximation curve $C_{fm}/C_{f0} = 1 - 0.65\,(M^2/Re)^{0.91}$; the notation for the experimental points is the same as in Fig. 6.

As the Reynolds number increases, the effect of suppression of the turbulent fluctuations gradually diminishes (for example, for $Re = 6.6 \cdot 10^4$ the fluctuation amplitudes diminish by 20-30%), and when $Re = 3 \cdot 10^5$ we no longer noted the pronounced effect of the magnetic field on the magnitude of the fluctuations in the total head.

An analysis of the oscillograms with respect to frequencies has not yet been completed and will evidently require additional testing. However, we can point to the preliminary conclusion that the field is more effective in connection with large-scale low-frequency fluctuations (5-30 Hz) than in connection with small-scale fluctuations of higher frequency (50-300 Hz). No fluctuation frequencies about 300 Hz were recorded in these experiments.

No changes in fluctuation frequency were observed as the amplitude diminished under the action of the magnetic field during the course of the experiments. In all of the cases, the maximum fluctuations were recorded at the point nearest the wall.

The qualitative pattern of the effect exerted by the magnetic field on the fluctuations in the static head is similar, although the phenomenon of fluctuation extinction is expressed to a smaller degree in this case. Processing of these oscillograms was complicated by the fact that the absolute magnitudes of the fluctuations in the static pressure are substantially lower than for the total head, so that for $Re = 5 \cdot 10^3$ the signal recorded from the sensor picking up the fluctuations in static head is commensurate with the noise level.

The friction factor was calculated from the formula $C_f = 2F/S\rho V^2$, where F is the experimentally determined force of friction on the channel walls swept by the mercury.

The results from the measurement of the coefficient of friction are shown in Fig. 5, from which we see that in the absence of a magnetic field the experimental values of C_f, within the investigated range of Reynolds numbers, are grouped rather well about the straight line given by the formula

$$C_{f0} = 0.013\,Re^{-0.25}. \tag{1}$$

It should be noted that the structure of (1) is reminiscent of the familiar Blasius formula for flow in a tube.

The curves $C_f = f(Re, M)$ have been drawn through the experimental points resulting from the mean statistical processing of the experimentally derived values of the friction force F. The error in the determination of C_f for $Re = 5 \cdot 10^3$ amounts to ±10%; for $Re \geq 10^4$ it does not exceed ±3%. Unfortunately, we were unable to investigate the region of transition from laminar to turbulent flow, because of the unstable motion of the cylinder and because of the great errors in the measurements as a consequence of the low velocities (less than 0.08 m/sec).

Figure 6 shows the reduction in the coefficient of friction at the wall as a function of the Hartmann number for various values of the Reynolds number. In processing these data, we found a good relationship between the ratio C_{fm}/C_{f0} and the Stewart number (Fig. 7). All of the points for which $M^2/Re < 1$ group well about the line described by the equation

$$C_{fm}/C_{f0} = 1 - 0.65\,(M^2/Re)^{0.91}. \tag{2}$$

The curves drawn through the experimental points in Fig. 5 virtually coincide with the indicated function, with the exception of several points for which $M^2/Re \geq 1$.

It is interesting to note that the experimental data of Genin and Zhilin [3] on the variation in the coefficient of friction as a function of the Re and M numbers for $M^2/Re \leq 1$ group themselves better about the curve $f(M^2/Re)$ than about the curve with the determining parameter M/Re, given in the cited reference.

Apparently, the entire region of variation in the coefficient of friction for the case of the turbulent flow of a conducting liquid in a magnetic field, beginning from Re_{cr}, can be divided into three zones:

1) when $M^2/Re \leq 1$ the relative friction factor (C_{fm}/C_{f0}) is a function of the Stewart number M^2Re;

2) when $M^2/Re > 1$, but with $M/Re < (2.5\text{-}3) \cdot 10^{-2}$, the relative coefficient of friction is a function of the parameter M/Re (see Fig. 5 in [3]);

3) when $M/Re > 3 \cdot 10^{-2}$ the coefficient of friction C_f is a function exclusively of the Reynolds number (see Fig. 3 in [3]).

It should be noted that the results of the Levin and Chinenkov [4] studies, corresponding to extremely high values of the Hartmann numbers (up to 2300), also show the presence of three zones of variation in the coefficient of friction. It is characteristic that λ/λ_T (the analog of our C_{fm}/C_{f0}) in this paper also ceases to be a function of the Hartmann number at $M/Re \geq 3 \cdot 10^{-2}$, while the points of flexure on the curves $\lambda/\lambda_T = f(M)$, which the authors propose to "take as the boundary defining the concept of weak and strong effects of the longitudinal magnetic field," are found at $M^2/Re \approx 5$.

Of course, we do not have adequate data for a solid determination of the "critical" values of M^2/Re and M/Re, nor to extend to all forms of flow the above statement as to the possibility of dividing the region of variation in the coefficient of friction into three zones. However, in our opinion, attention should be devoted to this question in subsequent studies.

NOTATION

$Re = V\delta/\nu$	is the Reynolds number;
$M = B_0\delta\sqrt{\sigma/\rho\nu}$	is the Hartmann number;
$Ta = \omega r^{0.5}\delta^{1.5}/\nu$	is the Taylor number;
V, ω)	are the linear and angular velocities for the rotating cylinder;
ρ, ν, σ	are, respectively, the density, the kinematic viscosity, and the electrical conductivity of the liquid;
r	is the radius of the rotating cylinder;
δ	is the clearance between the walls of the rotating cylinder and the channel;
B_0	is the mean integral magnetic induction in the channel;
$C_f = 2F/S\rho V^2$	is the coefficient of friction at the channel wall;
F	is the force of friction at the channel walls;
S	is the wetted area of the channel walls;
C_{f0}	is the coefficient of friction when $M = 0$;
C_{fm}	is the coefficient of friction when $M \neq 0$;
Re_{cr}	is the critical Reynolds number corresponding to the transition of laminar flow into turbulent flow.

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